The streets in many cities are organized using a grid system. In a grid system, the streets run parallel and perpendicular to each other, forming rectangular city blocks. You will use the Distance Formula to calculate the number of blocks between different locations in a city in which the streets are laid out in a grid system.
Quadrilateral \( TRAP \) is an isosceles trapezoid with \( TR \parallel PA \) and \( TR = RA \).

1. Plot the following points on the coordinate plane:
   \( R(6, 4) \)
   \( A(8, 0) \)
   \( P(0, 0) \)

   a. To complete the quadrilateral, what are the coordinates of point \( T \)?

   b. Describe how you located the coordinates of point \( T \).

   c. Locate point \( M \) on \( PA \) halfway between point \( P \) and point \( A \). What are the coordinates of point \( M \)?

   d. Describe how you located the coordinates of point \( M \).

   e. Locate point \( N \) on \( TP \) halfway between point \( T \) and point \( P \). What are the coordinates of point \( N \)?

   f. Describe how you located the coordinates of point \( N \).
2. If point $J$ is halfway between points $M$ and $D$, point $E$ is halfway between points $M$ and $T$, and point $E$ is halfway between points $G$ and $S$, what other information can you conclude from the figure shown?

Be prepared to share your methods and solutions.
Two friends, Shawn and Tamara, live in a city in which the streets are laid out in a grid system.

**PROBLEM 1**  Meeting at the Bookstore

Shawn lives on Descartes Avenue and Tamara lives on Elm Street as shown. The two friends often meet at the bookstore. Each grid square represents one city block.
1. How many blocks does Shawn walk to get to the bookstore?

2. How many blocks does Tamara walk to get to the bookstore?

3. Tamara wants to meet Shawn at his house so that they can go to a baseball game together. Tamara can either walk from her house to the bookstore and then to Shawn’s house, or she can walk directly to Shawn’s house. Which distance is shorter? Explain your reasoning.

4. Determine the distance, in blocks, Tamara would walk if she traveled from her house to the bookstore and then to Shawn’s house.

5. Determine the distance, in blocks, Tamara would walk if she traveled in a straight line from her house to Shawn’s house. Explain your calculation. Round your answer to the nearest tenth of a block.
6. Don, a friend of Shawn and Tamara, lives three blocks east of Descartes Avenue and five blocks north of Elm Street. Freda, another friend, lives seven blocks east of Descartes Avenue and two blocks north of Elm Street. Plot the location of Don’s house and Freda’s house on the grid. Label each location and label the coordinates of each location.

7. Another friend, Bert, lives at the intersection of the avenue that Don lives on and the street that Freda lives on. Plot the location of Bert’s house on the grid in Question 6 and label the coordinates. Describe the location of Bert’s house with respect to Descartes Avenue and Elm Street.

8. How do the coordinates of Bert’s house compare to the coordinates of Don’s house and Freda’s house?
9. a. Don and Bert often study French together. Use the house coordinates to write and evaluate an expression that represents the distance between Don’s and Bert’s houses.

b. How far, in blocks, does Don have to walk to get to Bert’s house?

10. a. Bert and Freda often study chemistry together. Use the house coordinates to write an expression that represents the distance between Bert’s and Freda’s houses.

b. How far, in blocks, does Bert have to walk to get to Freda’s house?

11. a. All three friends meet at Don’s house to study geometry. Freda walks to Bert’s house, and then they walk together to Don’s house. Use the coordinates to write and evaluate an expression that represents the distance from Freda’s house to Bert’s house and from Bert’s house to Don’s house.

b. How far, in blocks, does Freda walk altogether?
12. Draw the direct path from Don’s house to Freda’s house on the coordinate plane for Question 6. If Freda walks to Don’s house on this path, how far, in blocks, does she walk? Explain how you determined your answer.

13. Complete the summary of the steps that you took to determine the distance between Freda’s house and Don’s house. Let \( d \) be the direct distance between Don’s house and Freda’s house.

Distance between \( \text{Bert's house and Freda's house} \)  
\[
(\text{---} - \text{---})^2 + \text{---}^2 + \text{---}^2 = \text{---}
\]

Distance between \( \text{Don's house and Bert's house} \)  
\[
(\text{---} - \text{---})^2 + \text{---}^2 = \text{---}
\]

Distance between \( \text{Don's house and Freda's house} \)  
\[
\text{---}^2 = \text{---}
\]

Suppose Freda, Bert, and Don’s houses were at different locations. You can generalize their locations by using \( x_1, x_2, y_1, \) and \( y_2 \) and still solve for the distances between their houses.
14. Use the graph shown to determine the distance from:
   a. Don’s house to Bert’s house (DB)

   b. Bert’s house to Freda’s house (BF)

15. Use the Pythagorean Theorem to determine the distance from Don’s house to
    Freda’s house (DF).

You used the Pythagorean Theorem to calculate the distance between two points in
the plane. Your method can be written as the Distance Formula.

The **Distance Formula** states “If \((x_1, y_1)\) and \((x_2, y_2)\) are two points in the
coordinate plane, then the distance \(d\) between \((x_1, y_1)\) and \((x_2, y_2)\) is given by
\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.\]”

Indicate that distance is positive by using the absolute value symbol.

16. Do you think that it matters which point you identify as \((x_1, y_1)\) and which
    point you identify as \((x_2, y_2)\) when you use the Distance Formula? Explain your
    reasoning.
17. Calculate the distance between each pair of points. Round your answer to the nearest tenth if necessary. Show all your work.
   a. (1, 2) and (3, 7)  
   b. (−6, 4) and (2, −8)  
   c. (−5, 2) and (−6, 10)  
   d. (−1, −2) and (−3, −7)  

18. The distance between (x, 2) and (0, 6) is five units. Use the Distance Formula to determine the value of x. Show all your work.

Be prepared to share your methods and solutions.
Ms. Lopez is designing a treasure hunt for kindergarten children. The goal of the treasure hunt is for children to learn direction (right, left, forward, and backward) and introduce distance (near, far, in between). The treasure hunt will be on the school playground.
Ms. Lopez drew a model of the playground on a grid as shown. She uses this model to decide where to place items for the treasure hunt, and to determine how to write the treasure hunt instructions. Each grid square represents one square foot on the playground.

1. What are the coordinates of:
   a. the merry-go-round
   b. the slide
   c. the swings

2. Determine the distance, in feet, between the merry-go-round and the slide. Show all your work.

3. Ms. Lopez wants to place a small pile of beads in the grass halfway between the merry-go-round and the slide. How far, in feet, from the merry-go-round should the beads be placed? How far, in feet, from the slide should the beads be placed?
4. What should be the coordinates of the pile of beads? Explain how you determined your answer. Plot and label the pile of beads on the coordinate plane for Questions 1 through 4.

5. How do the coordinates of the pile of beads compare to the coordinates of the slide and merry-go-round?

6. Ms. Lopez also wants to place a pile of kazoos in the grass halfway between the slide and the swings. What should the coordinates of the pile of kazoos be? Explain your reasoning. Plot and label the pile of kazoos on the grid.

7. How do the coordinates of the pile of kazoos compare to the coordinates of the slide and swings?
8. Ms. Lopez wants to place a pile of buttons in the grass halfway between the swings and the merry-go-round. What do you think the coordinates of the pile of buttons will be? Explain your reasoning. Plot and label the pile of buttons on the shown grid.

9. How far, in feet, from the swings and the merry-go-round will the pile of buttons be? Show all your work and explain how you determined your answer. Round your answer to the nearest tenth if necessary.
10. Use the Distance Formula to determine whether your answer in Question 8 is correct by calculating the distance between the buttons and the swings. Show all your work.

   a. Would it have mattered if you verified your answer by calculating the distance between the buttons and the merry-go-round? Explain your reasoning.

11. Suppose the slide, the swings, and the merry-go-round were at different locations. You can generalize their locations by using $x_1$, $x_2$, $y_1$, and $y_2$.

   Solve for the location of the buttons.
   a. The vertical distance from the $x$-axis to the slide

   b. The distance from the slide to the swings
c. Half the distance from the slide to the swings

d. The vertical distance from the $x$-axis to the slide plus half the distance from the slide to the swings

e. Simplify the expression from part (d).

f. The horizontal distance from the $y$-axis to the slide

g. The distance from the slide to the merry-go-round

h. Half the distance from the slide to the merry-go-round

i. The horizontal distance from the $y$-axis to the slide plus half the distance from the slide to the merry-go-round

j. Simplify the expression from part (i).
The coordinates of the points that you determined in Question 11, part (e) and part (j), are midpoints, or points that are exactly halfway between two given points. The calculations you did in Question 11 can be summarized in the Midpoint Formula.

The **Midpoint Formula** states: “If \((x_1, y_1)\) and \((x_2, y_2)\) are two points in the coordinate plane, then the midpoint of the line segment that joins these two points is given by \(\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\).”

12. Determine the midpoint of each line segment that has the given points as its endpoints. Show all your work.
   a. \((0, 5)\) and \((4, 3)\)
   b. \((8, 2)\) and \((6, 0)\)
   c. \((-3, 1)\) and \((9, -7)\)
   d. \((-10, 7)\) and \((-4, -7)\)

Be prepared to share your methods and solutions.
OBJECTIVES
In this lesson you will:
- Determine whether lines are parallel.
- Identify and write the equations of lines parallel to given lines.
- Determine whether lines are perpendicular.
- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of horizontal and vertical lines.
- Calculate the distance between a line and a point not on the line.

KEY TERMS
- slope
- point-slope form
- slope-intercept form
- perpendicular
- reciprocal
- negative reciprocal
- horizontal line
- vertical line

Large parking lots, such as those located in a shopping center or at a mall, have line segments painted to mark the locations where vehicles are supposed to park. The layout of these line segments must be considered carefully so that there is enough room for the vehicles to move and park in the lot without the vehicles being damaged.

PROBLEM 1  Parking Spaces
Some line segments that form parking spaces in a parking lot are shown on the coordinate plane. One grid square represents one square meter.
1. What do you notice about the line segments that form the parking spaces?

2. What is the vertical distance between $AB$ and $CD$ and between $CD$ and $EF$?

3. Carefully extend $AB$ into line $p$, extend $CD$ into line $q$, and extend $EF$ into line $r$.

4. Use the graph to identify the slope of each line. What do you notice?

5. Use the point-slope form to write the equations of lines $p$, $q$, and $r$. Then write the equations in slope-intercept form.

6. What do you notice about the $y$-intercepts of these lines?

7. What do the $y$-intercepts tell you about the relationship between these lines?

8. If you were to draw a line segment above $EF$ to form another parking space, what would be the equation of the line that coincides with this line segment? Determine your answer without graphing the line. Explain your reasoning.
1. In the Parking Space problem, all the slopes were equal and the $y$-intercepts were all multiples of the same number.
   a. Are the slopes of parallel lines in a coordinate plane always equal? Explain.

   b. Are the $y$-intercepts of parallel lines in a coordinate plane always a multiple of the same number? Explain.

2. Write equations for three lines that are parallel to the line given by \( y = -2x + 4 \). Explain how you determined your answers.

3. Write an equation for the line that is parallel to the line given by \( y = 5x + 3 \) and passes through the point (4, 0). Show all of your work and explain how you determined your answer.

4. Without graphing the equations, determine whether the lines given by \( y - 2x = 5 \) and \( 2x - y = 4 \) are parallel. Show all of your work.
More Parking Spaces

Another arrangement of line segments that form parking spaces in a truck parking lot is shown on the grid. One grid square represents one square meter.

1. Use a protractor to determine the measures of \( \angle VUW \), \( \angle XWY \), and \( \angle ZYW \). What similarity do you notice about the angles?

2. Carefully extend \( \overline{UY} \) into line \( p \), extend \( \overline{UV} \) into line \( q \), extend \( \overline{WX} \) into line \( r \), and extend \( \overline{YZ} \) into line \( s \).

3. Name the perpendicular and parallel relationships. The relationships are:

4. Without actually determining the slopes, how will the slopes of the lines compare? Explain your reasoning.
5. What do you think must be true about the signs of the slopes of two lines that are perpendicular?

6. Use the graph and the lines you drew to determine the slopes of lines $p$, $q$, $r$, and $s$.

7. How does the slope of line $p$ compare to the slopes of lines $q$, $r$, and $s$?

8. What is the product of the slopes of two of your perpendicular lines?

When the product of two numbers is 1, the numbers are **reciprocals** of one another. When the product of two numbers is $-1$, the numbers are **negative reciprocals** of one another. So the slopes of perpendicular lines are negative reciprocals of each other.

9. Determine the negative reciprocal of each number.
   - a. 5
   - b. $-2$
   - c. $\frac{1}{3}$

10. Do you think that the $y$-intercepts of perpendicular lines tell you anything about the relationship between the perpendicular lines? Explain your reasoning.

11. Write equations for three lines that are perpendicular to the line given by $y = -2x + 4$. Explain how you determined your answers.
12. Write an equation for the line that is perpendicular to the line given by \( y = 5x + 3 \) and passes through the point \((4, 0)\). Show all your work and explain how you determined your answer.

13. Without graphing the equations, determine whether the lines given by \( y + 2x = 5 \) and \( 2x - y = 4 \) are perpendicular. Show all your work.

14. Describe the difference between the slopes of two parallel lines and the slopes of two perpendicular lines.

15. Suppose there is a line and you choose one point on the line. How many lines perpendicular to the given line can you draw through the given point?

16. Suppose there is a line and you choose one point that is not on the line. How many lines can you draw through the given point that are perpendicular to the given line? How many lines can you draw through the given point that are parallel to the given line?
1. What type of angles are formed by the intersection of the parking lot line segments? How do you know?

2. Carefully extend $\overline{LK}$ into line $p$, extend $\overline{GH}$ into line $q$, extend $\overline{FJ}$ into line $r$, and extend $\overline{KL}$ into line $s$.

3. Choose any three points on line $q$ and list their coordinates.

4. Choose any three points on line $r$ and list their coordinates.

5. Choose any three points on line $s$ and list their coordinates.

6. What do you notice about the $x$- and $y$-coordinates of the points you choose on each line?
7. What should be the equations of lines \( q, r, \) and \( s \)? Explain your reasoning.

8. Choose any three points on line \( p \).
   a. List their coordinates.
   
   b. What do you notice about the \( x \)- and \( y \)-coordinates of these points?
   
   c. What should be the equation of line \( p \)? Explain your reasoning.

In Problem 4, you wrote the equations of horizontal and vertical lines. A **horizontal line** has an equation in the form \( y = a \), where \( a \) is any real number. A **vertical line** has an equation in the form \( x = b \), where \( b \) is any real number.

9. Consider the horizontal lines you drew in Problem 4. For any horizontal line, if \( x \) increases by one unit, by how many units does \( y \) change?

10. What is the slope of any horizontal line? Explain your reasoning.

11. Consider the vertical line you drew in Problem 4. Suppose that \( y \) increases by one unit. By how many units does \( x \) change?

12. What is the rise divided by the run? Does this make any sense? Explain.

Because division by zero is undefined, a vertical line has an undefined slope.
13. Determine whether the statements are (always, sometimes, or never) true. Explain your reasoning.
   a. All vertical lines are parallel.

   b. All horizontal lines are parallel.

14. Describes the relationship between a vertical line and a horizontal line.

15. Write equations for a horizontal line and a vertical line that pass through the point (2, -1).

16. Write an equation of the line that is perpendicular to the line given by \( x = 5 \) and passes through the point (1, 0).

17. Write an equation of the line that is perpendicular to the line given by \( y = -2 \) and passes through the point (5, 6).

**PROBLEM 5**

**Distance Between Lines and Points**

1. Sketch a line and a point not on the line. Describe the shortest distance between the point and the line.
2. The equation of the line shown in the coordinate plane is \( f(x) = \frac{3}{2} x + 6 \). Draw the shortest segment between the line and the point \( A(0, 12) \). Label the point where the segment intersects \( f(x) \) as point \( B \).

3. What information is needed to calculate the length of \( \overline{AB} \) using the distance formula? Explain.

4. How can you calculate the intersection point of \( \overline{AB} \) and the line \( f(x) = \frac{3}{2} x + 6 \) algebraically?

5. Write an equation for \( \overline{AB} \).
6. Calculate the point of intersection of $\overline{AB}$ and the line $f(x) = \frac{3}{2}x + 6$.

7. Calculate the length of $\overline{AB}$.

8. What is the distance from the point $(0, 12)$ to the line $f(x) = \frac{3}{2}x + 6$?

9. Draw a line parallel to the line $f(x) = \frac{3}{2}x + 6$ that passes through the point $(0, 12)$. Identify another point on this parallel line.
10. Calculate the distance from the point you identified in Question 9 to the line \( f(x) = \frac{3}{2} x + 6 \).

11. Compare your answers to Question 8 and Question 10. Is this a coincidence? Explain.
12. Calculate the distance from the origin to the line $f(x) = \frac{3}{2}x + 6$.

13. Predict the distance from the point (2, 3) to the line $f(x) = \frac{3}{2}x + 6$. Explain.

Be prepared to share your methods and solutions.
In England, you can find flat, roughly circular areas of land that are enclosed by ditches, which are surrounded by piles of earth. These areas were created in ancient times and are called henges. The inner circular area of a henge can be accessed by entrances that were created through the surrounding ditches and piles of earth.
The circle on the coordinate plane represents the ditch of a henge. Each grid square represents a square that is three meters long and three meters wide.

1. Choose three points on the circle that will be the entrances to the inner area of the henge. Choose your points so that two of the points are endpoints of a diameter. Label the points $A$, $B$, and $C$ and include their coordinates.

2. Connect points $A$, $B$, and $C$ with line segments to form a triangle.

Your triangle is an inscribed triangle. An **inscribed triangle** is a triangle whose vertices lie on a circle.

3. Determine the slope of each side of your triangle. Show all your work.
4. Compare the slopes of the sides of the triangles. What do you notice?

5. Classify \( \triangle ABC \) by its angles.

6. Repeat Questions 1 through 5 for three different points named \( D, E, \) and \( F \). Show all of your work.

7. Write a conditional statement that describes the type of triangle that is created when the triangle is inscribed in a circle and one side of the triangle is a diameter.
8. Consider a triangle inscribed in a circle so that one side of the triangle is a diameter. Classify the side that is the diameter. Is this true for every triangle that is constructed in this way? Explain your reasoning.

9. Can you draw an inscribed right triangle in which none of the sides are a diameter? If so, name the vertices of this triangle.

**PROBLEM 2** Investigating Inscribed Triangles

1. The circle on the coordinate plane presents the ditch of a different henge. Each grid square represents a square that is two meters long and two meters wide. Choose three points on the circle that will be entrances to the inner area of the henge. Make sure that the points form the vertices of a right triangle. Label your points as points X, Y, and Z along with their coordinates on the coordinate plane.
2. Determine the coordinates of the midpoint of the hypotenuse. Label the midpoint on the graph as point $M$. Show all your work.

3. Calculate the distance from point $M$ to each of the vertices of the right triangle. Show all your work. Simplify, but do not evaluate any radicals.

4. What do you notice about the distances from the midpoint to the vertices?

5. Determine the midpoint coordinates for the other two sides of $\triangle XYZ$ in Question 1. Label these midpoints on the graph as points $N$ and $P$. 

---

**Take Note**

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points in the coordinate plane, then the midpoint of the line segment that joins these two points is given by 

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]
A **midsegment of a triangle** is a segment formed by connecting the midpoints of two sides of a triangle.

6. Points $N$ and $P$ are located on two sides of $\triangle XYZ$. Form a midsegment by connecting points $N$ and $P$.
   a. Determine the slope of the midsegment. How does the slope of the midsegment compare to the slope of the third side of $\triangle XYZ$?

   b. What can you conclude about midsegment $NP$ and the third side of $\triangle XYZ$? Consider the slopes of each.

   c. Calculate the length of midsegment $NP$ and the length of the third side of $\triangle XYZ$. 
d. What can you conclude about the relationship between the length of midsegment \(NP\) and the length of the third side of \(\triangle XYZ\)? Consider the lengths of each.

e. Repeat parts (a) through (d) using points \(M\) and \(P\).
f. Repeat parts (a) through (d) using points \( M \) and \( N \).


g. In summary, what can you conclude about the relationships between a midsegment of a triangle and the third side of the triangle?
1. The circle on the coordinate plane shown represents the ditch of a henge. Each grid square represents one square meter. The points \((-6, -8), (6, -8),\) and \((0, 10)\) represent entrances to the inner area of the henge. Label these three points.

2. Form a triangle by connecting the three points. Classify the triangle based on the lengths of its sides.

3. How can you verify the triangle classification from Question 2 using algebra?

4. Verify the triangle classification from Question 2 using algebra.
The Triangle Midsegment Theorem states: “The midsegment of a triangle is parallel to the third side of the triangle and half the measure of the third side of the triangle.”

1. Use the diagram to write the “Given” and “Prove” statements for the Triangle Midsegment Theorem.
   Given:
   Prove:

2. Create a two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Ms. Zoid asked her students to determine whether $\overline{RD}$ is a midsegment of $\triangle TUY$, given $TY = 14$ cm and $RD = 7$ cm.

Carson told Alicia that using the Triangle Midsegment Theorem, he could conclude that $\overline{RD}$ is a midsegment. Is Carson correct? How should Alicia respond if Carson is incorrect?

2. Ms. Zoid drew a second diagram on the board and asked her students to determine if $\overline{RD}$ is a midsegment of triangle $TUY$, given $\overline{RD} \parallel \overline{TY}$.

Alicia told Carson that using the Triangle Midsegment Theorem, she could conclude that $\overline{RD}$ is a midsegment. Is Alicia correct? How should Carson respond if Alicia is incorrect?
1. Use the Triangle Midsegment Theorem to determine the coordinates of the vertices of \( \triangle BJG \). Show all of your work.

2. Determine the perimeter of \( \triangle BJG \) and the perimeter of triangle \( FAR \). Round each radical to the nearest tenth. Show all of your work.
1. The vertices of triangle WHO are \( W(-2, 13) \), \( H(8, 3) \), and \( O(-5, 0) \). Is triangle WHO a scalene triangle, an isosceles triangle, or an equilateral triangle? Show all of your work.

Be prepared to share your solutions and methods.
What’s the Point?

Points of Concurrency

OBJECTIVES
In this lesson you will:

- Construct points of concurrency in triangles.
- Locate points of concurrency using algebra.

KEY TERMS
- concurrent
- point of concurrency
- incenter
- circumcenter
- median
- centroid
- orthocenter

PROBLEM 1
Concurrence

Concurrent lines, rays, or line segments are three or more lines, rays, or line segments intersecting at a single point. The point of concurrency is the point at which they intersect.

1. Draw three concurrent lines and label C as the point of concurrency.
2. Draw three concurrent rays and label $C$ as the point of concurrency.

3. Draw three concurrent line segments and label $C$ as the point of concurrency.
PROBLEM 2  Investigating the Incenter

1. Locate three points on the coordinate plane to form an acute triangle.

2. Connect the three points to form an acute triangle.

3. Construct the angle bisector of each angle in the acute triangle.

4. What do you notice about the intersection of the three angle bisectors?

5. What are the coordinates of the point of concurrency?
6. If the triangle drawn was an obtuse triangle, would the angle bisectors have a concurrent relationship? Use the coordinate plane to draw an obtuse triangle and construct the three angle bisectors.

The **incenter** is the point at which the three angle bisectors of a triangle are concurrent.

7. What can you conclude about the incenter?

8. Describe how to use the coordinate plane to prove your conclusions from Question 7.

**Take Note**
Recall that any point on an angle bisector is equidistant from the sides of the angle.
1. Locate three points on the coordinate plane to form an acute triangle.

2. Connect the three points to form an acute triangle.

3. Construct the perpendicular bisector of each side in the acute triangle.

4. What do you notice about the intersection of the perpendicular bisectors?

5. What are the coordinates of the point of concurrency?
6. If the triangle drawn was an obtuse triangle, would the perpendicular bisectors have a concurrent relationship? Use the coordinate plane to draw an obtuse triangle and construct the perpendicular bisectors of each side.

The circumcenter is the point at which the three perpendicular bisectors of the sides of a triangle are concurrent.

7. What can you conclude about the circumcenter?

8. Describe how to use the coordinate plane to prove your conclusions from Question 7.
The median of a triangle is a line segment formed by connecting a vertex of the triangle to the midpoint of the opposite side of the triangle.

1. Locate three points on the coordinate plane to form an acute triangle.

2. Connect the three points to form an acute triangle.

3. Construct the three medians of the triangle.

4. What do you notice about the intersection of the medians?

5. What are the coordinates of the point of concurrency?
6. If the triangle drawn was an obtuse triangle, would the medians have a concurrent relationship? Use the coordinate plane to draw an obtuse triangle and construct the three medians.

The centroid is the point at which the medians of a triangle are concurrent. The centroid is also referred to as a point of gravity or balance of the triangle. The entire triangle can be balanced on this point.

The centroid divides each median into two segments.

7. Choose one median and compare the length of the two segments. Compare the distance from the centroid to the vertex and the distance from the centroid to the opposite side. What is the ratio?

8. Does this ratio hold true for all three medians? Use the other two medians to make the same comparison.
1. Locate three points on the coordinate plane to form an acute triangle.

2. Connect the three points to form an acute triangle.

3. Construct three lines that contain the altitudes.

4. What do you notice about the intersection of the three lines containing the altitudes?

5. What are the coordinates of the point of concurrency?
6. If the triangle drawn was an obtuse triangle, would the lines containing the altitudes have a concurrent relationship? Use the coordinate plane to draw an obtuse triangle and construct three lines that contain the altitudes.

The orthocenter is the point at which the three altitudes, or lines containing the altitudes of a triangle are concurrent.
1. Form an isosceles triangle on the grid by connecting the points \((-6, 8), (6, -8),\) and \((0, 10)\).

2. Describe how you can locate the centroid of the triangle using two different methods.

3. Determine the coordinates of the centroid.

4. How can you locate the centroid of the triangle from Question 1 using algebra?
5. Use algebra to locate the centroid.

6. Compare the coordinates of the centroid using geometric tools in Question 3 and algebra in Question 5.
7. Use geometric tools to locate the circumcenter: in Question 1.

8. How can you locate the circumcenter of the triangle using algebra?

9. Use algebra to locate the circumcenter.
10. Compare the coordinates of the circumcenter using geometric tools in Question 6 and algebra in Question 8.

11. Label the centroid and circumcenter of the triangle on the coordinate plane in Problem 6, Question 1.

12. Use geometric tools to locate the orthocenter and the incenter of the triangle from Question 1. Label each on the grid. What do you notice about the four points of concurrency of an isosceles triangle?

PROBLEM 7  Equilateral Triangles and Points of Concurrency

1. Construct an equilateral triangle on the coordinate plane shown by completing the following steps.
   a. Plot points $A(-10, -10)$ and $B(10, -10)$. Draw $\overline{AB}$, one side of the equilateral triangle.

   b. Open the compass to a width equal to the distance between point $A$ and point $B$.

   c. Place the point of the compass on point $A$. Draw an arc above $\overline{AB}$.

   d. Keep the compass at the same width. Place the point of the compass on point $B$. Draw an arc above $\overline{AB}$. 
e. Label the intersection of the two arcs as point C. Draw $\overline{AC}$ and $\overline{BC}$.

2. What are the coordinates of point C?

3. Construct the incenter, circumcenter, centroid, and orthocenter. What do you notice about the four points of concurrency?

**Problem 8**

**Scalene Triangles in the Coordinate Plane**

1. Draw a large scalene triangle on the grid shown by plotting three points labeled D, E, and F and connecting the points.
2. Identify the coordinates of the vertices of the scalene triangle.

3. Show that the triangle is scalene using geometric tools.

4. Show that the triangle is scalene using algebra.

5. Construct the incenter, circumcenter, centroid, and orthocenter of \( \triangle DEF \). Which of these points of concurrency are collinear?
Determine which point of concurrency is most useful in each situation. Explain your reasoning.

1. A small town wants to find a new location for its firehouse. The new location must be equally accessible to all three main roads shown.

2. Valerie wants to center a table at a location equidistant from her refrigerator, stove, and sink.

Be prepared to share your solutions and methods.
Lesson 9.6
Planning a Subdivision
Quadrilaterals in a Coordinate Plane

OBJECTIVES
In this lesson you will:
- Classify quadrilaterals in a coordinate plane.
- Prove the Trapezoid Midsegment Theorem.

KEY TERMS
- midsegment of a trapezoid
- Trapezoid Midsegment Theorem

PROBLEM 1
The Lay of the Land

A land planner is laying out different plots, or parcels of land for a new housing subdivision. The parcels of land will be shaped like quadrilaterals.

Parcel 1 is shown on the coordinate plane. Each grid square represents one square acre.
1. What kind of quadrilateral is parcel 1?

2. Find the slope of each side of the parcel.
   a. How many pairs of opposite sides, if any, are parallel? Explain your reasoning.

   b. Are any of the sides perpendicular? Explain how you know.

   c. Classify the quadrilateral with the information you have so far.

3. Find the lengths of the sides that form parcel 1. Show all your work. Are any of the side lengths congruent? If so, describe the sides that are congruent.

4. Can you classify parcel 1 further? If so, classify the quadrilateral.

Take Note
Remember that the Distance Formula is \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).
You can use this formula to find the lengths of the sides that form parcel 1.
5. The endpoint coordinates of parcel 2 are \(E(5, 1), F(5, 6), G(9, 6),\) and \(H(9, 1).\) Graph parcel 2 on the coordinate plane. Classify this quadrilateral in as many ways as is possible. Explain your reasoning.

6. Calculate the lengths of the diagonals. What do you notice?
The endpoint coordinates of parcel 3 are $J(5, 6)$, $K(7, 10)$, $L(11, 10)$, and $M(9, 6)$. Graph parcel 3 on the coordinate plane. Classify this quadrilateral in as many ways as is possible. Explain how you found your answer.
1. A land developer is creating parcels of land in the shape of parallelograms. Plot the points \(Q(-3, -5), R(2, -5), \) and \(S(3, -3)\) on the coordinate plane shown. Each square represents one square acre.

2. Locate a fourth point \(T\) such that the points \(Q, R, S,\) and \(T\) form a parallelogram. What are the coordinates of point \(T\)?

3. How can you verify that the quadrilateral is a parallelogram using algebra?

4. Use an algebraic strategy to verify the quadrilateral is a parallelogram.
5. Calculate the area of the parcel of land defined by the parallelogram in Question 4.

6. Plot the points $Q(-3, -5), R(2, -5), \text{ and } S(3, -3)$ on the coordinate plane shown.

7. Locate a fourth point $T$ that is different than the point in Question 2 and connect the four points to form a parallelogram. What are the coordinates of point $T$?

8. Use an algebraic strategy to verify the quadrilateral is a parallelogram.
9. Calculate the area of the parcel of land defined by the parallelogram in Question 8.

Take Note
Remember that the height of the parallelogram is a perpendicular line segment that connects a vertex to the opposite side.

10. Did you calculate the same area in Questions 5 and 9? Why do you think this result occurred?

11. Molly claims there is another possible location for point T such that the points Q, R, S, and T form a parallelogram. Is she correct? Explain.
1. Graph points \((2, 12), (-2, 4), (4, 0),\) and \((8, 8)\).

2. Connect the four points to form a quadrilateral.

3. Calculate the midpoints of the diagonals of this quadrilateral using the midpoint formula.

4. What do you notice about the midpoints of the diagonals?

5. Confirm your conclusion in Questions 3 and 4 using your graph. Does the graph support your conclusion?

6. Classify the quadrilateral.
1. Draw a square on the coordinate plane.

2. What geometric figure is determined by connecting the midpoints of the sides of a square? Use your graph, the Midpoint Formula, and the Distance Formula in your answer.

3. Graphically, compare the area of the new figure with the area of the original square. What could you conclude?
1. Locate four points on the coordinate plane to form a trapezoid.

2. What are the coordinates of the four points?

3. Determine the coordinates of the midpoints of the legs of the trapezoid. Use the midpoint formula.

4. Plot and connect the midpoints of the legs. Determine the distance between the two midpoints.

The midsegment of a trapezoid is a segment formed by connecting the midpoints of the legs of the trapezoid.

5. Determine the lengths of the two bases of the trapezoid.
6. Determine the length of the midsegment of the trapezoid.

7. Compare the length of the midsegment to the sum of the lengths of the bases.

8. Is the midsegment of the trapezoid parallel to the bases of the trapezoid? Explain.

**PROBLEM 6  Trapezoid Midsegment**

The **Trapezoid Midsegment Theorem** states: “The midsegment of a trapezoid is parallel to each of the bases and its length is one half the sum of the lengths of the bases.”

1. Prove the Trapezoid Midsegment Theorem. It will be necessary to connect points \( M \) and \( E \) to form \( ME \) and then extend \( ME \) until it intersects the extension of \( DS \) at point \( T \).
Hint: First prove $\triangle MEG \cong \triangle TES$ and then show $JE$ is the midsegment of $\triangle MDT$.

**a.** Complete the “Prove” statement.

Given: $MDSG$ is a trapezoid

- $J$ is the midpoint of $MD$
- $E$ is the midpoint of $GS$

**b.** Create a two-column proof of the Trapezoid Midsegment Theorem.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
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Be prepared to share your solutions and methods.
Chapter 9 Checklist

KEY TERMS
- Distance Formula (9.1)
- midpoint (9.2)
- Midpoint Formula (9.2)
- slope (9.3)
- point-slope form (9.3)
- perpendicular (9.3)
- reciprocal (9.3)
- negative reciprocal (9.3)
- horizontal line (9.3)
- vertical line (9.3)
- slope-intercept form (9.3)
- perpendicular (9.3)
- inscribed triangle (9.4)
- midsegment of a triangle (9.4)
- concurrent (9.5)
- point of concurrency (9.5)
- incenter (9.5)
- circumcenter (9.5)
- median (9.5)
- centroid (9.5)
- orthocenter (9.5)
- horizontal line (9.3)
- vertical line (9.3)
- inscribed triangle (9.4)
- midsegment of a trapezoid (9.6)

THEOREMS
- Triangle Midsegment Theorem (9.4)
- Trapezoid Midsegment Theorem (9.6)

CONSTRUCTIONS
- angle bisector (9.5)
- incenter (9.5)
- perpendicular bisector (9.5)
- circumcenter (9.5)
- median (9.5)
- centroid (9.5)
- altitude (9.5)
- orthocenter (9.5)
- equilateral triangle (9.5)

9.1

Using the Distance Formula

You can use the Distance Formula to calculate the distance between two points in the coordinate plane. The Distance Formula states that if \((x_1, y_1)\) and \((x_2, y_2)\) are two points in the coordinate plane, then the distance \(d\) between \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

Example:
To calculate the distance between the points \((3, -2)\) and \((-5, 1)\), use the Distance Formula.

\[
x_1 = 3, \ y_1 = -2, \ x_2 = -5, \ y_2 = 1
\]

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(-5 - 3)^2 + (1 - (-2))^2}
\]

\[
= \sqrt{(-8)^2 + 3^2}
\]

\[
= \sqrt{64 + 9}
\]

\[
= \sqrt{73} \approx 8.5
\]

So, the distance between the points \((3, -2)\) and \((-5, 1)\) is \(\sqrt{73}\) units, or approximately 8.5 units.
Using the Midpoint Formula

A midpoint is a point that is exactly halfway between two given points. You can use the Midpoint Formula to calculate the coordinates of a midpoint.

The Midpoint Formula states that if \((x_1, y_1)\) and \((x_2, y_2)\) are two points in the coordinate plane, then the midpoint of the line segment that joins these two points is given by \[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

Example:
To calculate the midpoint of a line segment whose endpoints are \((-8, -3)\) and \((4, 6)\), use the Midpoint Formula.

\[
x_1 = -8, \ y_1 = -3, \ x_2 = 4, \ y_2 = 6
\]

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-8 + 4}{2}, \frac{-3 + 6}{2} \right) = \left( \frac{-4}{2}, \frac{3}{2} \right) = \left( -2, \frac{3}{2} \right)
\]

So, the midpoint of the line segment whose endpoints are \((-8, -3)\) and \((4, 6)\) is \((-2, \frac{3}{2})\).

Determining Whether Lines are Parallel or Perpendicular

When two lines are parallel, their slopes are equal. When two lines are perpendicular, their slopes are negative reciprocals of each other. When the product of two numbers is \(-1\), the numbers are negative reciprocals of each other.

Examples:
The equation of line \(p\) is \(y = 2x + 6\), the equation of line \(q\) is \(y = 2x - 10\), and the equation of line \(r\) is \(y = -\frac{1}{2}x\). The slope of line \(p\) is 2, the slope of line \(q\) is 2, and the slope of line \(r\) is \(-\frac{1}{2}\). The slopes of lines \(p\) and \(q\) are equal, so lines \(p\) and \(q\) are parallel. The slopes of lines \(p\) and \(r\) are negative reciprocals of each other, so lines \(p\) and \(r\) are perpendicular. Also, the slopes of lines \(q\) and \(r\) are negative reciprocals of each other, so lines \(q\) and \(r\) are also perpendicular.
Determining the Distance Between Lines and Points

To determine the distance between a line and a point not on the line, follow the steps below.

- Determine the equation of the perpendicular segment that is drawn from the given point to the given line. Write the equation in slope-intercept form.
- Calculate the point of intersection of the given line and the perpendicular segment.
- Use the Distance Formula to calculate the distance between the point of intersection and the given point.

Examples:
The distance between the line $y = \frac{4}{3}x + 2$ and the point $(−4, 5)$ can be calculated as follows.

Equation of perpendicular segment: $y = mx + b$

$5 = -\frac{3}{4}(-4) + b$
$5 = 3 + b$
$2 = b$
$y = \frac{3}{4}x + 2$

Point of intersection:

$\frac{4}{3}x + 2 = -\frac{3}{4}x + 2$
$16x = -9x$
$25x = 0$
$x = 0$
$y = \frac{4}{3}(0) + 2 = 2$

$(0, 2)$

Distance between point of intersection and given point:

$x_1 = 0, y_1 = 2, x_2 = -4, y_2 = 5$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-4 - 0)^2 + (5 - 2)^2}$
$= \sqrt{(-4)^2 + 3^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25} = 5$

So, the distance between the line $y = \frac{4}{3}x + 2$ and the point $(−4, 5)$ is 5 units.
Determining Midsegments of Triangles

A midsegment of a triangle is a segment formed by connecting the midpoints of two of its sides.

Example:
To determine the coordinates of the midsegments of the triangle shown, calculate the midpoints of the sides of the triangle.

Midpoint of \( XY \) = \( \left( \frac{-5 + (-5)}{2}, \frac{6 + (-4)}{2} \right) = (-5, 1) \)

Midpoint of \( YZ \) = \( \left( \frac{-5 + 9}{2}, \frac{6 + (-4)}{2} \right) = (2, 1) \)

Midpoint of \( XZ \) = \( \left( \frac{-5 + 9}{2}, \frac{-4 + (-4)}{2} \right) = (2, -4) \)

Segments \( AB, BC, \) and \( AC \) are the midsegments of \( \triangle XYZ \).
Using the Triangle Midsegment Theorem

The Triangle Midsegment Theorem states: “The midsegment of a triangle is parallel to the third side of the triangle and half the measure of the third side of the triangle.”

Example:
Triangle $ABC$ is inscribed in circle $D$. Segments $DE$, $EF$, and $DF$ are midsegments of $\triangle ABC$. So, the following statements are true:

$AB \parallel DF$ $BC \parallel DE$ $AC \parallel EF$

$DF = \frac{1}{2} AB$ $DE = \frac{1}{2} BC$ $EF = \frac{1}{2} AC$
Identifying Points of Concurrency

When three or more lines intersect at the same point, the lines are called concurrent lines. The point at which the concurrent lines intersect is called the point of concurrency.

There are special types of points of concurrency in triangles. The incenter of a triangle is the point at which the three angle bisectors of a triangle are concurrent.
The circumcenter of a triangle is the point at which the three perpendicular bisectors of a triangle are concurrent. The centroid is the point at which the three medians of a triangle are concurrent. The orthocenter is the point at which the three altitudes of a triangle are concurrent.

Examples:
In \( \triangle ABC \), \( \overline{AE} \), \( \overline{BF} \), and \( \overline{CD} \) are angle bisectors.
So, point \( G \) is the incenter, and \( DG = EG = FG \).

In \( \triangle DEF \), \( \overline{JK} \), \( \overline{LM} \), and \( \overline{NP} \) are perpendicular bisectors. So, point \( Q \) is the circumcenter, and the distances from the circumcenter to each vertex are the same.

In \( \triangle PQR \), \( \overline{PT} \), \( \overline{QV} \), and \( \overline{RS} \) are medians.
So, point \( W \) is the centroid, and \( PW = 2TW \), \( QW = 2VW \), and \( RW = 2SW \).

In \( \triangle XYZ \), \( \overline{XB} \), \( \overline{YC} \), and \( \overline{ZA} \) are altitudes.
So, point \( D \) is the orthocenter.
9.6

Classifying Quadrilaterals in the Coordinate Plane

To classify quadrilaterals in the coordinate plane, use the formula for the slope of a line, the Distance Formula, properties of slope and parallel lines, and properties of slope and perpendicular lines.

Examples:

Quadrilateral $ABCD$:
The coordinates are $A(2, 0), B(0, 8), C(8, 10),$ and $D(10, 2)$.

Slope of $AB = \frac{8 - 0}{0 - 2} = -4$;

Slope of $BC = \frac{10 - 8}{8 - 0} = \frac{1}{4}$;

Slope of $CD = \frac{2 - 10}{10 - 8} = -4$;

Slope of $DA = \frac{2 - 0}{10 - 2} = \frac{1}{4}$

$AB = \sqrt{(0 - 2)^2 + (8 - 0)^2} = \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$

$BC = \sqrt{(8 - 0)^2 + (10 - 8)^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$

$CD = \sqrt{(10 - 8)^2 + (2 - 10)^2} = \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$

$DA = \sqrt{(2 - 10)^2 + (0 - 2)^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$

The opposite sides of $ABCD$ have the same slope, so the opposite sides are parallel. The adjacent sides of $ABCD$ have slopes that are negative reciprocals of each other, so the adjacent sides are perpendicular. All four sides of $ABCD$ have the same length. So, $ABCD$ is a square.

Quadrilateral $JKLM$:
The coordinates are $J(-7, -7), K(-4, -2), L(3, -2),$ and $M(0, -7)$.

Slope of $JK = \frac{-2 - (-7)}{-4 - (-7)} = \frac{5}{3}$; Slope of $KL = \frac{-2 - (-2)}{3 - (-4)} = \frac{0}{7} = 0$;

Slope of $LM = \frac{-7 - (-2)}{0 - 3} = \frac{5}{3}$; Slope of $MJ = \frac{-7 - (-7)}{-7 - 0} = \frac{0}{-7} = 0$

$JK = \sqrt{(-4 - (-7))^2 + (-2 - (-7))^2} = \sqrt{9 + 25} = \sqrt{34}$;

$KL = \sqrt{3 - (-4))^2 + (-2 - (-2))^2} = \sqrt{9} = 3$;

$LM = \sqrt{(0 - 3)^2 + (-7 - (-2))^2} = \sqrt{9 + 25} = \sqrt{34}$;

$MJ = \sqrt{(-7 - 0)^2 + (-7 - (-7))^2} = \sqrt{9} = 3$

The opposite sides of $JKLM$ have the same slope, so the opposite sides are parallel. The adjacent sides of $JKLM$ do not have slopes that are negative reciprocals of each other, so the adjacent sides are not perpendicular. The sides of $JKLM$ do not have the same length. So, $JKLM$ is a parallelogram.
9.6 Using the Trapezoid Midsegment Theorem

The midsegment of a trapezoid is a segment formed by connecting the midpoints of the legs of the trapezoid.

The Trapezoid Midsegment Theorem states: “The midsegment of a trapezoid is parallel to each of the bases and its length is one half the sum of the lengths of the bases.”

Examples:
Segment $XY$ is the midsegment of trapezoid $ABCD$. So, the following statements are true:

- $XY \parallel AD$
- $XY \parallel BC$
- $XY = \frac{1}{2}(AD + BC)$

![Diagram of a trapezoid with midsegment $XY$ connecting midpoints of legs $AD$ and $BC$.]